John von Neumann 1903-1957

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If influence of a scientist is interpreted broadly enough to include impact on fields beyond science proper then John von Neumann was probably the most influential mathematician ever lived: not only did he contribute to almost all branches of modern mathematics and created new fields but he also changed history after the second World War by his work in computer design and by being a sought-after technical advisor to the post-war military-political establishment of the U.S.A. To celebrate John von Neumann's 100th birthday, the international 'Von Neumann Centennial Conference" took place in Budapest, Hungary between October 15-20, 2003. Part of this event was the "Linear operators and foundations of quantum mechanics" conference, where von Neumann's legacy in operator theory was reviewed and discussed by leading experts in this field. During the conference the American Mathematical Society and the János Bolyai Mathematical Society unveiled a commemorative plaque on the house in Budapest where von Neumann was born and raised. To remember von Neumann the present note sketches von Neumann's life and career and recalls briefly some of his views on the nature of mathematics.

1 Childhood and Education

John von Neumann (known in Hungary as Neumann János) was born in 1903 in Hungary to a well-to-do family, wealth of which was established by his father, Max von Neumann, a successful banker, during the calm and economically prosperous years of the Austro-Hungarian Monarchy that followed the so-called "self rule" in 1867 that secured Hungary's semi-independence within the Monarchy. Accordingly, von Neumann had a first rate education. This education started by home schooling and included language instruction in the form of presence of German speaking maids in the household in downtown Budapest, where von Neuman was raised. German being von Neumann's first second language, his German remained superior to his English until about the mid thirties. When time came, von Neumann enrolled in the famous, expensive, private Protestant high school in Budapest. His talent in mathematics was recognized there by László Rátz, von Neumann's his high school mathematics teacher. Rátz asked for (and got) permission from von Neumann's father to arrange tutoring von Neumann in mathematics by faculty members of the Technical University in Budapest, insisting at the same time that von Neumann attend regular mathematics classes, which von Neumann did. As a result of this private tutoring von Neumann had been prepared already in high school to become a professional mathematician; yet, after graduating from high school the von Neumann's made the decision to enroll John von Neumann in the chemical engineering program of the Eidgenössiche Technische Hochschule (ETH) in Zürich, Switzerland. Chemical engineering was a very popular field at the time; in addition, a chemical engineer had a far better chance of landing a job than did a mathematician, a consideration that weighed heavily in the eyes of von Neumann's father, a practically minded person. However, simultaneously with his education at ETH, von Neumann also studied mathematics in Berlin and in Budapest, and he finished his formal university studies by receiving his PhD in mathematics (axiomatic set theory) in Budapest.

2 Career

In 1926 von Neumann went to Göttingen on a Rockefeller fellowship to work as Hilbert's assistant. Göttingen was not only one of the centers of mathematics but it also was a mecca of theoretical physics; thus in Göttingen von Neumann could familiarize himself with the latest developments concerning quantum mechanics. Hilbert himself gave lectures on the mathematical foundations of quantum mechanics in the academic year 1926-1927. Von Neuman attended these lectures, and working out the lecture notes taken during those lectures led to a joint publication [4] and eventually to von Neumann's three ground breaking papers [9, 10, 11] on the mathematical foundations of quantum mechanics that served as the basis of his book [12].

After a short stay in Berlin and Hamburg as non-tenured faculty ("Privatdozent"), he was invited by Princeton University in 1929 to lecture. In January 1930 he was offered a permanent professorship in Princeton University, which von Neumann declined; he then was entrusted to substitute for the Jones professorship in mathematical physics for five years. Finally, in January 1933, he accepted the invitation to become one of the first six permanent professors of the Institute of Advanced Study (IAS) established in Princeton in 1933.

Contrary to the still widespread belief that von Neumann had left Europe for fear of political prosecution, von Neumann was keen on emphasizing that he had taken up residency in the USA before the political situation in Europe became unbearable and that he therefore never considered himself a refugee scientist (von Neumann's letter to M.R. Davie (May 3, 1946) [17]). In harmony with his being a non-refugee scientist at IAS von Neumann was regularly visiting Europe – and Hungary in particular – during the thirties; it was only in the face of imminent threat of war that he decided not to visit Europe any more.

Von Neumann retained his academic position in Princeton until the end of his life, but he spent a lot of time by travelling and lecturing in different universities and research institutes. During the war von Neumann became increasingly involved in military-related research, among other things he participated in the Manhattan Project. After the war von Neumann's military and governmental consulting activity expanded both in volume and in significance tremendously: he was serving on a number of very influential committees that shaped post-war U.S. military policy. At the peak of his power he lists the following appointments as the most significant advisory positions: consultant to the U.S. Atomic Energy Commission; consultant to the Armed Forces Special Weapons Project; member of the Scientific Advisory Board of the U.S. Air Force; member of the Scientific Advisory Committee of a classified Air Force project connected with the Office of the Special Assistant (Research and Development) of the Secretary of the Air Force; member of the Technical Advisory Board on Atomic Energy connected with the Office of the Assistant Secretary of Defense (Research and Development) and member of the Scientific Advisory Committee of the U.S. Army Ordnance Corps, Ballistic Research Laboratories, Aberdeen Proving Ground (letter to colonel H.H. Rankin, August 26, 1954, [17]). His "exceptionally outstanding service" during the war to the United States and its Navy was acknowledged by awarding him the *Medal for Merit* (October, 1946) and the *Distinguished Service Award* (July, 1946).

After the war von Neumann received offers from different universities, especially from MIT and UCLA. MIT's offer included the promise to fund an electronic computer project at MIT, making available MIT's extensive engineering know-how. Computer design was in the focus of von Neumann's academic interest after the war and he was urging IAS' leadership to make IAS the home of an electronic computer development project. His efforts an persistency paid off: IAS, with support from Radio Corporation of America (RCA) and Princeton University decided to carry out a computer project with von Neumann as director. Having secured IAS' support for the computer project von Neumann declined MIT's offer – but he did this with great regret, anticipating what later became increasingly obvious, namely that IAS' was not the ideal place to carry out the engineering-heavy computer project. It is fair to say more generally that IAS was not the most suitable institution for von Neumann after the war: IAS' ivory-tower-like intellectual climate was not entirely ready to accommodate von Neumann's increasingly application oriented interests; in addition, von Neumann always had a very extensive and diverse consulting activity not only in government but also in the private sector (he was consultant to the Standard Oil Company and IBM for instance), which does not seem to have been welcomed by IAS either. Von Neumann must have realized this while he was on official leave from IAS as Atomic Energy Commissioner from 1954 because he did not intend to return to IAS after his job as Atomic Energy Commissioner: he was again offered a position at MIT in 1956 and simultaneously he also was negotiating a position at UCLA. Von Neumann finally decided in March 1956 to accept the offer to be appointed as professor at large at UCLA;

he was never able to take the position however because he died of incurable cancer on February 8, 1957 in Washington D.C. He is buried in the Princeton cemetery.

Given the unparalleled diversity and depth of von Neumann's contribution to both pure and applied mathematics it is impossible to list even his major achievements in a short biographic sketch. Rather than trying to do the impossible, I am recalling below von Neumann's general views about mathematics and its relation to the physical sciences.

3 Von Neumann's views on the nature mathematics

3.1 Von Neumann's interpretation of Gödel's incompleteness theorems

Von Neumann started his career as a mathematician with work on axiomatic set theory and he also worked on Hilbert's program aimed at proving consistency of mathematics by finitistic means. The turning point in the history of Hilbert's program was the famous Second Conference for Epistemology of the Exact Sciences that took place between 5 and 7 of September 1930 in Königsberg. It was in the discussion session on September 7 of this conference that Gödel announced the first version of what became known as Gödel's first incompleteness theorem: every sufficiently rich and consistent axiom system contains meaningful statements that are undecidable within that system. Von Neumann grasped immediately the significance of Gödel's result for the axiomatic foundation of mathematics, pressed Gödel for further details and, as his letter to Gödel (November 20, 1930) [2],[17] shows, shortly after the Königsberg conference, he apparently obtained what is known as Gödel's second incompleteness theorem. The second incompleteness theorem says that the consistency of a sufficiently rich axiomatic theory cannot be proved within the system itself: the statement expressing consistency of the system in the system is an undecidable proposition in the system. Von Neumann informed Gödel of this result, only learning that Gödel himself had already established this consequence of the first incompleteness result. Von Neumann then acknowledged Gödel's priority and did not publish anything on the topic. (For a history of the Königsberg conference see [1],[3] and the references therein.)

From the second incompleteness theorem von Neumann had drawn a very strong conclusion for the Hilbert program: "... there is no rigorous justification for classical mathematics" (letter of von Neumann to Gödel, November 29, 1930). On this point von Neumann strongly disagreed with Gödel: in his letter to Carnap (June 7, 1931) ([3] and [17]) von Neumann writes:

> Thus I am today of the opinion that
>
> 1. Gödel has shown the unrealizability of Hilbert's program
> 2. There is no more reason to reject intuitionism (if one disregards the aesthetic issue, which in practice will also for me be the decisive factor).
>
> Therefore I consider the state of the foundational discussion in Königsberg to be outdated, for Gödel's fundamental discoveries have brought the question to a completely different level. (I know that Gödel is much more careful in the evaluation of his results, but in my opinion on *this* point he does not see the connections correctly).

3.2 Von Neumann on mathematical rigor

The unrealizability of Hilbert's program was a decisive development shaping von Neumann's views on the nature of mathematics: in his view this development showed that there is no immovable notion of rigor in mathematics and that one cannot justify classical mathematics by mathematical means:

> Whatever philosophical or epistemological preferences anyone may have in this respect, the mathematical fraternities' actual experiences with its subject give little support to the assumption of the existence of an a priori concept of mathematical rigor.
>
> I have told the story of this controversy [*debate about the foundations of mathematics*] in such detail, because I think that it constitutes the best caution against taking the immovable rigor of mathematics too much for granted. This happened in our lifetime, and I know myself how humiliatingly easily my own views regarding the absolute mathematical truth changed during this episode, and how they changed three times in succession! [14][p. 6]

> It is *not* necessarily true that the mathematical method is something absolute, which was revealed from on high, or which somehow, after we got hold of it, was evidently right and has stayed evidently right ever since. To be more precise, maybe it *was* evidently right after it was revealed, but it certainly didn't stay evidently right ever since. There have been very serious fluctuations in the professional opinion of mathematicians on what mathematical rigor is. To mention one minor thing: In my own experience, which extends over only some thirty years, it has fluctuated so considerably, that my personal and sincere conviction as to what mathematical rigor is, has changed at least twice. And this is in a short time of the life of one individual! [13][p. 480]

> The variability of the concept of rigor shows that something else besides mathematical abstraction must enter into the makeup of mathematics. [14][p. 4]

The "something else" is empirical science, physics in particular:

> ... some of the best inspirations of modern mathematics (I believe, the best ones) clearly originated in the natural sciences. [14][p. 2]

Von Neumann mentions geometry and analysis as examples of mathematical disciplines that clearly have empirical origins but he firmly believed that all mathematical disciplines have an empirical origin, however remote one, and that mathematics' becoming detached from its empirical roots carries with it a risk:

> As a mathematical discipline travels far from its empirical source . . . it is beset with very grave dangers. It becomes more and more purely aestheticizing, more and more purely *l'art pour l'art*. [14][p. 9].

The field is then in danger of developing along the line of least resistance and will "separate into a multitude of insignificant branches" [14][p. 9].

> Whenever this stage is reached, the only remedy seems . . . to be a rejuvenating return to the source: the reinjection of more or less directly empirical ideas. [14][p. 9].

But the relation of mathematics and sciences is a two-way one: that sciences fertilize mathematics is just one aspect of their rich mutual dependence. The other side of their relationship is that mathematics also permeates science:

> The most vitally characteristic fact about mathematics is, in my opinion, its quite peculiar relationship to the natural sciences, or, more generally, to any science which interprets experience on a higher than purely descriptive level. [14][p. 1]
>
> In modern empirical sciences it has become a major criterion of success whether they have become accessible to the mathematical method or to the near-mathematical methods of physics. Indeed, throughout the natural sciences an unbroken chain of pseudomorphoses, all of them pressing toward mathematics, and almost identified with the idea of scientific progress, has become more and more evident. [14][p. 2]

4 Von Neumann on the axiomatic method in physics

In harmony with his relaxed attitude about mathematical rigor in mathematics von Neumann also took a moderate position about mathematical precision in physics. Specifically, he saw that the axiomatic method cannot be practiced in physics the way it can in mathematics: von Neumann embraced what is dubbed in [7] "opportunistic soft axiomatization" (see also [6] and [8]). Its explicit formulation can be found already in the 1926 joint paper by Hilbert, Nordheim and von Neumann on the foundations of quantum mechanics [4]. This paper contains a relatively lengthy passage on the axiomatic method in physics. The main idea is that a physical theory consists of three, sharply distinguishable parts: (i) physical axioms, (ii) analytic machinery (also called "formalism") and (iii) physical interpretation.

The physical axioms are supposed to be semi-formal requirements (postulates) formulated for certain physical quantities and relations among them. The basis of these postulates is our experience and observations. The analytic machinery is a mathematical structure containing quantities that have the same relation among themselves as the relation between the physical quantities. *Ideally*, the physical axioms should be strong and rich enough to *determine* the analytic machinery *completely.* The physical interpretation connects then the elements of the analytic machinery and the physical axioms. But the ideal situation never occurs (hence the terminology "opportunistic soft axiomatization"):

> In physics the axiomatic procedure alluded to above is not followed closely, however; here and as a rule the way to set up a new theory is the following.
>
> One typically conjectures the analytic machinery before one has set up a complete system of axioms, and then one gets to setting up the basic physical relations only through the interpretation of the formalism. It is difficult to understand such a theory if these two things, the formalism and its physical interpretation, are not kept sharply apart. This separation should be performed here as clearly as possible although, corresponding to the current status of the theory, we do not want yet to establish a complete axiomatics. What however is uniquely determined, is the analytic machinery which – as a mathematical entity – cannot be altered. What can be modified – and is likely to be modified in the future – is the physical interpretation, which contains a certain freedom and arbitrariness. [4][p. 106], translation form [7].

A closer look at how von Neuman actually treated and presented quantum mechanics reveals that he did indeed follow the methodology of opportunistic soft axiomatization in his work on quantum theory [7].

One may wonder what made von Neumann so successful not only in pure mathematics but in a wide variety of other disciplines as well. While there is no easy and simple answer to this question, it seems plausible that his exceptional talent was combined with a broad education that avoided narrow-minded concentration on mathematics and this made him appreciative and receptive of the problems of the real world.

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